Quiz 1

- Please write your section number and your name on the booklet
- Answer each problem on a seperate page of the booklet.

1. (10 points) Given a telescoping series

$$S = \sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} = \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right).$$

   Let s_N denote the Nth partial sum of S. First find a simple formula for s_N in terms of N. Then determine how much should N be to guarantee that $\mid S - s_N \mid \leq 0.01$.

2. (20 points) Determine whether the following series converge or diverge. Be sure to indicate what test you are using and carry out all work related to that test.

   - $\sum_{n=1}^{\infty} \frac{2+cosn}{n^2}$.
   - $\sum_{n=1}^{\infty} \frac{n-1}{\sqrt{n^3-1}}$.
   - $\sum_{n=1}^{\infty} \left(\frac{5n+lnn}{8n-lnn}\right)^n$.

- $\sum_{n=1}^{\infty} \frac{n+3}{3^n}$.

3. (20 points) Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{lnn}$ converges absolutely, converges conditionally, or diverges. Be sure to indicate what test you are using and carry out all work related to that test. Also estimate the error in using the 5th partial sum to approximate the total sum.

4. - (10 points) Find the radius and interval of convergence of the power series
$$\sum_{n=1}^{\infty} \frac{n(x-1)^n}{4^n}.$$
   - (10 points) Find only the radius of convergence of the power series
$$\sum_{n=0}^{\infty} \frac{n!(n+2)!}{(2n)!} x^n$$
.

5. (20 points) Given that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges for all x. Find the power series in powers of x for $f(x) = \int e^x dx$ and determine its radius of convergence. Do you get the series of e^x, explain.

6. (10 points)

   - Show that $tan^{-1}x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ for $\mid x \mid< 1$.
   - Find also the Taylor polynomial $p_1(x)$ generated by $f(x) = tan^{-1}x$ at $x = 0$. Then use the alternating series estimation theorem to estimate the error resulting from the approximation $tan^{-1}(-0.1) \sim p_1(-0.1)$.

   Hint: The derivative of $f(x) = tan^{-1}x$ is $\frac{1}{1+x^2}$